Synopsis of Proposed Research Work

A Study on Fundamental Group Structures in the Context of Soft Set Setting

Candidate's Name

Injamam Ul Karim

Department of Mathematics
Siksha Bhavana
Visva-Bharati
Santiniketan-731235
West Bengal, India

Supervisor: Prof Swapan Raha,
Deparment of Mathematics,
Siksha Bhavana
Visva-Bharati

Co- Supervisor: Dr. Nazmul Sk,
Department of Mathematics,
Bankura University
Bankura-722155

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1. Introduction

In dealing with the uncertainties inherent in the problems of physical science, biological science, engineering, economics, social science, medical science etc, various theories such as probability theory, fuzzy set theory, rough set theory, soft set theory were developed. Probability theory deals with the uncertainties associated with the randomness of the situation. But there are some kind of uncertainties evolving out of imprecise information, limitations of computability and perceptibility etc. In 1965, L. A. Zadeh attempted to formalize this grey area by introducing the concept of fuzzy set where the passage from 'yes' to 'no' is gradual not sharp. Later several generalizations and modifications of fuzzy sets came up with various perspectives. To mention some of these are Lattice valued fuzzy set by J.A.Goguen in 1967, Rough set by Pawlak in 1982, Intuitionistic fuzzy set by Atanassov in 1986, Interval-valued fuzzy set by M.B. Gorzalzany in 1987, Vague set by W.L.Gau and D.J.Buehrer in 1993, etc. In fuzzy set theory crucial role is played by the membership function and finding the right choice of membership function depends on various factors. In 1999, Molodstov tried to approach in an alternative way. Instead of taking the membership function he took a parameterized family of sets and called it "Soft set". Recently mathematical activities are being conducted with all these concepts individually as well as their hybridizations.

2. Brief history of previous work done:

After the invention of the theory of fuzzy sets by Zadeh in 1965, many related concepts such as L-fuzzy set, intuitionistic fuzzy set, intuitionistic L-fuzzy set, fuzzy subgroups, fuzzy topology, fuzzy topological groups etc. were developed. In 1967, J. A. Goguen [16] published the theory of L-fuzzy sets. In 1982, Z. Pawlak [39] introduced Rough set theory. This is a set with imprecise boundaries. It is basically an approximate representation of a given crisp set in terms of two subsets of a crisp partition defined on the universal set involved. In 1983 - 1986, K. Atanasov [2, 4] gave the idea of Intuitionistic fuzzy sets and intuitionistic L-fuzzy set, a set with each member having a degree of belongingness as well as a degree of non-belongingness. This is again a generalization of fuzzy set theory. In 1971 Rosenfeld [40] first introduced the idea of fuzzy subgroup and Chang [6], Wong [42], Lowen [20] and others applied some basic concepts from general topology to fuzzy sets and developed a theory of fuzzy topological spaces and Foster [15], YU and MA [9,10] on fuzzy topological groups. Recently in 1999, Molodtsov [25] proposed a new approach, viz. Soft set theory for

modelling vagueness and uncertainty in a system. In 2001-2003, P.K.Maji, R.Biswas and A.Roy ([21], [22], [23], [24]) have done a theoretical study on soft sets and fuzzy soft sets. After that research activities are going on different algebraico-topological structures in fuzzy sets, soft sets and their hybridizations. In view of this Aktas and Cagman [1] in 2007, introduced soft groups which was extended fuzzy soft groups [29], intuitionstic L-fuzzy soft groups [32] by Nazmul and Samanta to develop algebraic topological structures in fuzzy and soft settings. Also in 2011, Muhammad Shabir and Munazza Naz [27] introduced the theory of soft topology. Afterwards in 2010-2015, Nazmul and Samanta have introduced different types of topological group structures in the context of fuzzy and soft settings such as soft topological groups [28], soft topological soft groups [30], fuzzy topological groups [31], fuzzy soft topological groups [32], group soft topology [33], generalised group soft topologies [34] etc and study their properties.

The objective is to develop soft fundamental group structures, study homomorphism and isomorphism of different subsystems of such structures. This research is significant as soft set theory have rich potential for applications in diverse fields which include among others social, engineering and medical sciences.

3. Proposed plan of work :

After reviewing the previous works on topological groups and related field in the context of fuzzy and soft set settings it is seen that researchers have started working in the fields of fuzzy groups, soft groups, fuzzy soft groups, fuzzy topology, soft topology, fuzzy topological group, soft topological groups, soft topological soft groups, fuzzy soft topological groups etc. On the other hand, fundamental group is an important algebraic-topological structure. But from the literature surveyed above, it appears that there is ample scope for future research to develop fundamental group structure in soft set settings.

More specifically in my Ph.D. research work I would like to study in the following directions.

(1) Introduce different algebraic and topological structures such as path, loop, homotopy, graphs and free groups in handling the soft type uncertainties. For example soft path, soft loop, soft homotopy, soft free groups, soft covering space, soft homology and other hybrid structures formed from these.

(2) Study the corresponding algebraico-topological properties where topological concepts also involve softness or other such hybrid structures required to deal with uncertainties.

(3) Study subsystems of these structures.

(4) Study homomorphisms and isomorphisms of these structures.

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